Lafayette Problem Group

Thursday, September 18, 2003

Next Meeting: Thursday, September 25.

Problem 1: If n is an integer and n is not prime, explain why n must divide $(n-1)!$.

Problem 2: Three men robbed another man of a vase containing 24 ounces of oil. Whilst running away, they met a merchant selling glass containers, from whom they purchased three vessels. Upon reaching a place of safety, they wished to divide their booty, but they found that the vessels held 5, 11, and 13 ounces, respectively. How can they divide the oil evenly?

Problem 3: Find a polynomial with integer coefficients that has the number $\sqrt{2}+\sqrt{3}$ as a root.

Problem 4: Can you find three points in the plane that form an equilateral triangle if the coordinates of the points are required to be integers?

Problem 5: Suppose a and b are both irrational numbers. Show that either $a+b$ or $a-b$ must be irrational.

Problem 6: Here's a simple process that generates a sequence $a_1, a_2, a_3, \ldots$ of integers. Pick a_1 to be any positive integer you wish. Then, generate the sequence by the following rules. If a_i is even, let $a_{i+1} = a_i/2$. If a_i is odd, let $a_{i+1} = 3a_i + 1$. For example, if you set $a_1 = 10$, then the sequence generated is $10, 5, 16, 8, 4, \ldots$.

Does any sequence generated in this way have to contain the number 1?

Problem 7: Color every point of $\mathbb{R}^2$ red, blue, or green. Prove that there are two points of the same color that are one unit apart.