Lafayette Problem Group

Everyone is welcome to join! This semester we will valiantly try as a group to solve problems published in mathematics journals.

Meetings are Fridays at Lunchtime in Pardee 216 – BYOLunch

From Mathematics Magazine, due May 1, 2009

Problem 1806: The intersection of the ellipsoid $x^2 + y^2 + \frac{z^2}{c^2} = 1$ and the plane $x + y + cz = 0$ is an ellipse. For $c > 1$, find the value of c for which the area of the ellipse is maximal.

From The College Mathematics Journal, due April 15, 2009

Problem 893: Let f be any function that has a Taylor series representation at 0 with radius of convergence 1, and let

$$T_n(x) = f(0) + \frac{f'(0)}{1!}x + \cdots + \frac{f^{(n)}(0)}{n!}x^n$$

denote the nth-degree Taylor polynomial of f. Find the sum

$$\sum_{n=1}^{\infty} x^n (f(x) - T_n(x))$$

for $|x| < 1$.

From The American Mathematical Monthly, due April 30, 2009

Problem 11397: Let a, b, c, x, y, z be positive numbers such that $a+b+c = x + y + z$ and $abc = xyz$. Show that if $\max\{x, y, z\} \geq \max\{a, b, c\}$, then $\min\{x, y, z\} \geq \min\{a, b, c\}$.

Problem 11401: Let A be a nonsingular square matrix with integer entries. Suppose that for every positive integer k, there is a matrix X with integer entries such that $X^k = A$. Show that A must be the identity matrix.

Problem 11402: Let $f : [0, 1] \to [0, \infty)$ be a continuous function such that $f(0) = f(1) = 0$ and $f(x) > 0$ for $0 < x < 1$. Show that there exists a square with two vertices in the interval $(0, 1)$ on the x-axis and the other two vertices on the graph of f.