

## Lafayette Problem Group – Problem Set 5

*Everyone is welcome! Try to get solutions, or good ideas, or even just bad ideas, for some of these problems by next week's meeting:*

**Thursday, October 9**  
**Lunchtime in Pardee 216**

Let's begin with a few problems for which the technique of *mathematical induction* might help...

**Problem 1:** Determine a formula for

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)}$$

and explain why your formula must be correct for all natural numbers  $n$ .

**Problem 2:** Explain why  $11^n - 6$  is divisible by 5 for all positive integers  $n$ .

**Problem 3:** Determine which amounts of postage can be made using only 5-cent and 7-cent stamps.

**Problem 4:** What is the maximum number of different regions into which  $n$  lines can split the plane?

**Problem 5:** *The LVAIC Exam will take place in early November, and you can participate – more details soon! Here's a problem from a recent exam.*

Suppose  $a$  and  $b$  are positive real numbers. Show that  $a(1-b)$  and  $b(1-a)$  cannot both be greater than  $1/4$ .

**Problem 6:** *Here's a problem from a recent Putnam exam. This year's Putnam Exam will be on Saturday, December 9. Sign up in Pardee 218 by October 9!*

Given any five points on a sphere, show that some four of them must lie on a closed hemisphere. ("Closed" just means that the hemisphere includes its boundary.)

**Remember to visit [www.lafayette.edu/~math/](http://www.lafayette.edu/~math/)!**