

Lafayette Problem Group – Problem Set 4

Everyone is welcome! Try to get solutions, or good ideas, or even just bad ideas, for some of these problems by next week's meeting:

Thursday, September 25
Lunchtime in Pardee 216

Problem 1: You roll an ordinary die (a cube with the numbers 1 - 6) repeatedly and keep track of the running sum. You stop rolling as soon as this running sum is greater than 24. What is the most likely value of the sum when you stop? (*Unanswered from last week.*)

Problem 2: On a circle, n points are selected, and then the chords joining them in pairs are drawn. Assuming that no three chords intersect in a single point (except at the endpoints), how many points of intersection are there?

Problem 3: Find positive numbers n and a_1, a_2, \dots, a_n such that

$$a_1 + a_2 + \dots + a_n = 1000$$

and the product $a_1 a_2 \dots a_n$ is as large as possible.

Problem 4: Show that the fraction

$$\frac{21n + 4}{14n + 3}$$

is irreducible for every positive integer n .

Problem 5: *The LVAIC Exam will take place this fall, and you can participate – more details soon! Here's a problem from a recent exam.*

Which number is larger, 19^{99} or 99^{19} ? Explain carefully.

Problem 6: *Here's a problem from a recent Putnam exam.*

Show that if the points of an isosceles right triangle of side length 1 are each colored with one of four colors, then there must be two points of the same color which are at least a distance $2 - \sqrt{2}$ apart. (*Unanswered from last week.*)

Remember to visit www.lafayette.edu/~math/!