

## Lafayette Problem Group

Thursday, October 6, 2005

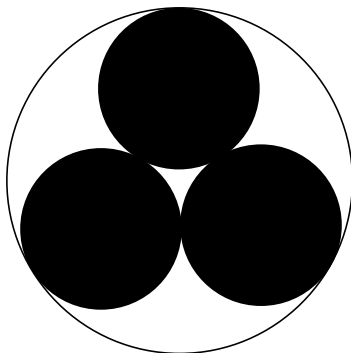
4:15 in Pardee 227

*Try as many of these as you can by next week's meeting! Good luck!*

**Problem 1:** What are all possible values of positive integers  $a, b, c$  and  $n$  such that

$$n^a + n^b = n^c ?$$

**Problem 2:** If the outer circle has radius 1, what is the value of the shaded area?



**Problem 3:** Let  $a, b$  and  $\phi$  be real numbers, and let  $x, y$  and  $z$  be rational numbers such that

$$\begin{aligned}x &= a + b \cos(\phi) \\y &= a + b \cos\left(\phi - \frac{2\pi}{3}\right) \\z &= a + b \cos\left(\phi + \frac{2\pi}{3}\right)\end{aligned}$$

and  $27ab^2 = 4$ , then  $(x, y, z)$  is a permutation of  $(1, 0, 0)$ .

*(Over)*

**Problem 4:** Show that for any function  $f$  with continuous second derivative on  $[0, 1]$  and with  $f(0) = f(1) = 0$ , the following inequality holds:

$$2 \int_0^1 (f'(x))^2 dx \leq 2005 \int_0^1 (f(x))^2 dx + \frac{1}{2005} \int_0^1 (f''(x))^2 dx.$$

**Problem 5:** This problem comes from a former Putnam exam, a national competition offered the first Saturday of every December. This year's exam will take place from 10am until 6pm on Saturday, December 3. Sign up by contacting Professor Smith at [smithder@lafayette.edu](mailto:smithder@lafayette.edu).

Let  $A$  be a positive real number. What are the possible values of  $\sum_{j=0}^{\infty} x_j^2$ , given that  $x_0, x_1, \dots$  are positive numbers for which  $\sum_{j=0}^{\infty} x_j = A$ ?

**Problem 6:** Get the solution to this problem before October 31 and we can submit it for publication in the *American Mathematical Monthly*.

Show that for all integers  $n \geq 3$ , the number of *compositions* of  $n$  into relatively prime parts is a multiple of 3. A composition of  $n$  into  $k$  parts is a list of  $k$  positive integers that sum to  $n$ . Thus, there are six compositions of 4 into relatively prime parts:

$$(3, 1), (1, 3), (2, 1, 1), (1, 2, 1), (1, 1, 2), (1, 1, 1, 1).$$

**Remember to visit [www.lafayette.edu/~math](http://www.lafayette.edu/~math) often!**