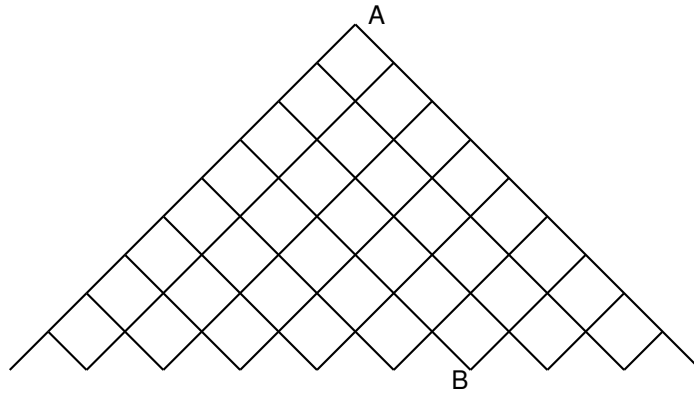


## Lafayette Problem Group

Thursday, September 8, 2005  
4:15 in Pardee 227

*Try as many of these as you can by next week's meeting! Good luck!*

**Problem 1:** How many paths are there from point  $A$  to point  $B$  that use exactly 9 edges?



**Problem 2:** (a) A circle of radius 1 is in the first quadrant of the plane and is tangent to both the  $x$ -axis and the  $y$ -axis. Find the radius of the largest circle that fits into the corner.

(b) A ball of radius 1 is in a corner of a room touching all three walls. Find the radius of the largest ball that fits into the corner.

**Problem 3:** 2 million different points in the plane  $\mathbb{R}^2$  are chosen randomly. True or false: there is a line  $\ell$  that separates exactly 1 million of the points from the others.

*(Over)*

**Problem 4:** On a table is a row of fifty coins, of various denominations. Alice picks a coin from one of the ends and puts it in her pocket; then Bob chooses a coin from one of the (remaining) ends, and the alternation continues until Bob pockets the last coin.

Prove that Alice can play so as to guarantee at least as much money as Bob.

**Problem 5:** This problem comes from a former Putnam exam, a national competition offered the first Saturday of every December. This year's exam will take place from 10am until 6pm on Saturday, December 3.

A *composite* (positive integer) is a product  $ab$  with  $a$  and  $b$  not necessarily distinct integers in  $\{2, 3, 4, \dots\}$ . Show that every composite is expressible as  $xy + xz + yz + 1$ , with  $x$ ,  $y$ , and  $z$  positive integers.

**Problem 6:** Get the solution to this problem before December 31 and we can submit it for publication in the *American Mathematical Monthly*.

Let  $e_n$  and  $o_n$  be the number of dissections of a convex  $n$ -gon by nonintersecting diagonals into an even or odd number of regions, respectively. Show that  $e_n - o_n = (-1)^n$ .

**Remember to visit [www.lafayette.edu/~math](http://www.lafayette.edu/~math) often!**