Problem Set No. 5
Due Friday, March 2, 2001

Read: Rowell and Wormley, pp.169–187, study the figures. Read pp. 120-124.

Transformers and Transducers

In dynamic system models, transformers and transducers are interfaces which transmit power between subsystems. They are the dynamic system elements that permit us to model useful systems since most machines have a combination of subsystems. The terms "transformer" and "transducer" have specific definitions in system dynamics, which differ from, but are based on, their common engineering usage.

The term "transformer" commonly refers to an electrical transformer, which transforms electrical power at the interface between two electrical subsystems. An ideal electrical transformer neither dissipates nor stores energy. Deferring the question of signs for the time being by using absolute values, we can express this statement of energy conservation in terms of power flow into and out of the transformer as:

$$|\mathbf{P}_{in}| = |v_{in} i_{in}| = |\mathbf{P}_{out}| = |v_{out} i_{out}|$$

Using electrical transformers as a model, in system dynamics terminology, a "transformer" interfaces subsystems of the same type of energy. Examples of transformers include:

- Levers interface two translational mechanical subsystems.
- Gear sets interface two rotational mechanical systems.
- Belt drives interface two rotational mechanical systems.

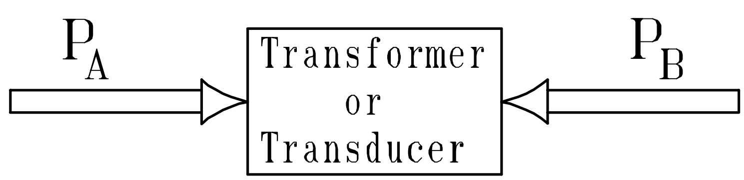
The term "transducer" commonly refers to a sensor, such as a load cell, which emits an electrical signal in response to a non-electrical input, a force in the case of a load cell. Consequently, a sensor interfaces two dissimilar energy systems. Using a sensor as a model, in system dynamics terminology, transducers interface subsystems of different types of energy. There is a significant difference between a sensor and a system dynamics transducer. Sensors produce signals, which are information, not power. An ideal sensor's signal is a time varying voltage or current, not power, which is the product of current and voltage. (Real signals require a finite amount of power to be transmitted and processed.) Likewise, the mechanical power needed to produce a force signal from a load cell is extremely small, since a load cell actually senses elastic displacements, which, given the size of a load cell, are very small.

System dynamic transducers are similar to transformers in that they interface the power flow between two subsystems. They differ from transformers because they interface dissimilar subsystems. Examples of transducers include:

- DC motors interface electrical and rotational mechanical subsystems.
- Hydraulic pistons interface fluid and translational mechanical subsystems.
- Racks and pinions interface translational and rotational mechanical subsystems.
- Pumps interface fluid and rotational or translational mechanical systems.

Real transformers and transducers have dynamic properties in addition to the ability to transform or transduce power. For example, a real lever has mass, compliance, and friction in addition to its transformer property. Similarly, a real motor or generator stores kinetic energy in its rotational inertia and magnetic energy in the magnetic field created by the current through its windings. It also dissipates energy as friction in its bearings and electrical resistance its contacts and conductors. In a dynamic model, each significant energetic property must be represented by a different element. If the mass of a real lever is significant to the dynamic performance of a system, then a mass element **and** a transformer element must both be included in the lumped parameter dynamic model to represent both the energy storage and transformation properties of the lever. If the rotational inertia and friction is significant in a real electric motor then an inertia element, a damping element, and a transducer element must be included to represent these three independent energetic properties.

The transformation or transduction of power across an interface between subsystems can be represented schematically as a rectangle, or "block", with **two power flows both shown flowing into it**, each entering an energy "port". Transformers and transducers are "two port elements". A two port element can have energy flow through it, entering one port and exiting the other. Our passive elements and sources are one port elements. Energy must flow into or out of the element by a single "port". We will use the "checkbook" sign convention for power flow. Power flow into a port is positive. Power flow out is negative.



Because a transformer or transducer neither stores nor dissipates energy, power which flows in one side must flow out the other. Consequently, although both flows are shown flowing in, one must flow out. Power can flow in either direction across the interface. In other words, either $\mathbf{P}_A$ or $\mathbf{P}_B$ can be $\mathbf{P}_{in}$ at a given instant. Showing both flows **into** the transformer or transducer underlines this fact. It does introduce an awkward sign in circumstances in which we believe we know the direction of power flow across the interface because the assumption of both power flows into the interface necessitates one of the flows being negative. Although signs will be a nuisance in some cases, assuming both flows are into the interface is a useful, general purpose assumption, particularly in systems in which power can oscillate back and forth across the interface.

Transformer and transducer equations are derived beginning with energy conservation expressed in terms of power flows:

$$\mathbf{P}_A + \mathbf{P}_B = 0$$

which can be rearranged as:

$$\mathbf{P}_A = -\mathbf{P}_B$$

This expression represents the power flow into the two ports of the interface, but it does not provide the equations we need to derive a differential system equation. We have two power variables on each side of the transformer or transducer interface for a total of four variables. Say we were to work with an electrical transformer where:

$$\mathbf{P}_A = i_A v_{1g} \quad \text{and} \quad \mathbf{P}_B = i_B v_{2g}$$

Expressing energy conservation in terms of power yields:

$$i_A v_{1g} + i_B v_{2g} = 0$$

This equation can be rearranged to express one variable in terms of the other three, say:

$$i_B = -\left(\frac{i_A v_{1g}}{v_{2g}}\right)$$

Although it is a valid equation, is not a particularly useful addition to our equation list because we derive a system equation by substituting to eliminate unwanted power variables. This equation would introduce three variables in place of one.

We prefer equations which contain only two variables, so that we do not add variables to the equation upon substitution. Consequently, instead of a single equation with four variables, we will derive two equations formulated in terms of a power variable from each side of the interface of the form:

$$v_{2g} = n\, v_{1g}$$

and

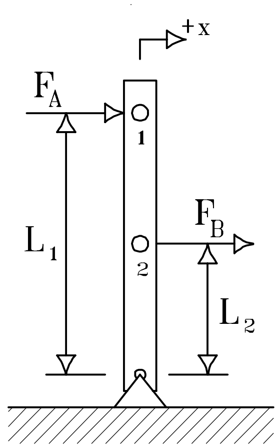
$$i_B = -\left(\frac{1}{n}\right) i_A$$

where n is the “transformer” constant. We will include these equations in our list of equations extracted from the linear graph.

We will illustrate the linear graph technique beginning with mechanical transformers, levers and gears. We will then extend your ME 332 characterization of the relationship between motor current and torque as an electromechanical transducer, a DC motor.

Mechanical Transformers: Levers and Gears.

We will assume that the rotation and resulting horizontal displacement of the lever shown below is small, so we do not have to consider motion in the vertical direction. Within this assumption, the lever is a translational mechanical transformer. Further, we will assume that the only property of the device is transformation; the lever does not store or dissipation energy. For this to be reasonably true, the lever would need low mass (small kinetic energy storage), high stiffness (small strain energy storage), and low friction at the pivot (small energy dissipation). Define an arbitrary positive direction, as shown. Assume that there are two translational velocity nodes on the lever, nodes 1 and 2, and show forces F_A and F_B acting at nodes 1 and 2 in the assumed positive direction.



Expressing energy conservation in terms of power for the lever, recognizing that the power $\mathbf{P}_A$ flowing into or out of the lever at node 1 is:

$$\mathbf{P}_A = F_A v_{1g}$$

and the power $\mathbf{P}_B$ flowing into or out of the lever at node 2 is:

$$\mathbf{P}_B = F_B v_{2g}$$

if

$$\mathbf{P}_A = -\mathbf{P}_B$$

then

$$F_A v_{1g} = -F_B v_{2g}$$

If the lever can be modeled as rigid, then the translational velocity of nodes 1 and 2 are related. The translational velocities are both functions of the angular velocity of the lever Ω and the distance of the node from the pivot:

$$v_{1g} = L_1 \Omega$$

and

$$v_{2g} = L_2 \Omega$$

We can relate v_{1g} and v_{2g}:

$$\frac{v_{1g}}{L_1} = \Omega = \frac{v_{2g}}{L_2}$$

Hence,

$$v_{2g} = \left(\frac{L_2}{L_1} \right) v_{1g}$$

Having established a relationship between v_{1g} and v_{2g} from the geometry of the lever, we can now substitute into the equation of energy conservation expressed in terms of power to determine a relationship between F_A and F_B:

$$F_A v_{1g} = -F_B \left(\frac{L_2}{L_1} \right) v_{1g}$$

$$F_B = -\left(\frac{L_1}{L_2} \right) F_A$$

The "transformer equations" for this lever are:

$$v_{2g} = \left(\frac{L_2}{L_1} \right) v_{1g} \qquad \text{and} \qquad F_B = -\left(\frac{L_1}{L_2} \right) F_A$$

Note that there is a fractional and sign inversion in the constant term between these two equations. The fractional inversion is a result of energy conservation, $|\mathbf{P}_A| = |\mathbf{P}_B|$. The sign inversion is a result of our convention to assume both $\mathbf{P}_A$ and $\mathbf{P}_B$ flow into the transformer, $\mathbf{P}_A + \mathbf{P}_B = 0$. The sign inversion between F_A and F_B tells us that one of the forces is acting in the negative direction.

The fractional and sign inversion between the two equations is independent of the specifics of the transformer or transducer. Renaming the constant term:

$$n \equiv \left(\frac{L_2}{L_1} \right)$$

we can write:

$$v_{2g} = n\, v_{1g} \qquad \text{and} \qquad F_B = -\left(\frac{1}{n} \right) F_A$$

All transformers and transducer equations have a fractional and sign inversion of the transformer or transducer constant when they are written with the power variable from one side of the interface on the same side in both equations. In the transformer equations above, the power variables for $\mathbf{P}_B$, v_{2g} and F_B, are on the left side of both equations.

We did not need to derive the transformer equation using the relationship between the velocities of nodes 1 and 2. If we had preferred, we could have used a statement of moment equilibrium about the pivot. Defining counterclockwise as positive, with the forces acting in the positive x-direction, as shown:

$$F_A L_1 + F_B L_2 = 0$$

which leads to:

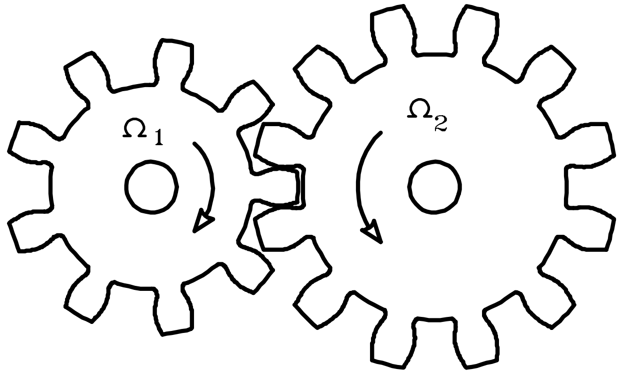
$$F_B = -\left(\frac{L_1}{L_2} \right) F_A$$

It is generally preferable to use geometric compatibility of velocities rather than equilibrium of forces or moments, when possible, for two reasons. First, equilibrium equations are more prone to sign errors of action versus reaction. It is easier to see the direction of a velocity than a force or moment. Second, it is often more difficult to formulate the equilibrium statement than the geometric compatibility statement. Our lever analysis is a good example. Note that summation of the horizontal forces shown is not a statement of equilibrium:

$$F_A + F_B \neq 0$$

because we did not include the horizontal reaction force at the pivot. That is why we wrote a moment equilibrium statement about the pivot. It was obvious in this case that we were missing the reaction force, but there are many mechanisms where the relevant equilibrium condition or all of the reaction forces and moments may not be as obvious.

Consider the two intermeshing spur gears shown in the schematic below. A positive direction for the angular velocities and shaft torques must be defined vectorally using the right-hand rule. The pinion rotates at angular velocity Ω_{1g} and the gear rotates at Ω_{2g}. Torque T_A acts on the pinion shaft and torque T_B on the gear shaft, both in the positive direction.



Proceeding as with the lever, power $\mathbf{P}_A$ flowing into or out of the pinion is:

$$\mathbf{P}_A = T_A \Omega_{1g}$$

and the power $\mathbf{P}_B$ flowing into or out of the gear is:

$$\mathbf{P}_B = T_B \Omega_{2g}$$

if

$$\mathbf{P}_A + \mathbf{P}_B = 0$$

then

$$\mathbf{P}_A = -\mathbf{P}_B$$

or

$$T_A \Omega_{1g} = -T_B \Omega_{2g}$$

We can see from the schematic of the gears that they rotate in opposite directions (counter-rotate). Both gears must have the same surface velocity of the pitch surface so that the teeth mesh. The smaller pinion must rotate faster than the larger gear to feed teeth into the nip at the same rate. The equilibrium condition in the transformer relationship is somewhat counter-intuitive. Levers have moment equilibrium about the pivot. One can think of the teeth of meshed spur gears as a series of levers making and breaking contact with one another. The contact force between the meshed gear teeth (the tooth force) must be equal and opposite (action and reaction). The torque about the respective shafts are not equal and opposite, unless the gears have the same pitch radius.

Pitch radius is an inconvenient dimension to measure. It is far easier to use the number of teeth on the circumference to express the relative size of a gear, since the teeth on both gears must be the same size for them to mesh. The relative surface speeds and shaft torques of the gears can be expressed in terms of the ratio of the number of teeth, since the radius of a circle equals the circumference divided by 2π. Consequently, the angular velocities of the two gears can be related to their pitch surface speed by their respective teeth numbers:

$$v_{surface} = r_p \Omega_{1g} = -r_G \Omega_{2g}$$

Using w_{tooth}, the width of a tooth, and the teeth numbers N_1 and N_2 to calculate the circumferences:

$$\left(\frac{N_1 w_{tooth}}{2\pi}\right)\Omega_{1g} = -\left(\frac{N_2 w_{tooth}}{2\pi}\right)\Omega_{2g}$$

which simplifies to:

$$N_1 \Omega_{1g} = -N_2 \Omega_{2g}$$

or:

$$\Omega_{1g} = -\left(\frac{N_2}{N_1}\right)\Omega_{2g}$$

We can now substitute back into the equation for energy conservation expressed in terms of power to derive the relationship between T_A and T_B:

$$T_A \Omega_{1g} = -T_B \Omega_{2g}$$

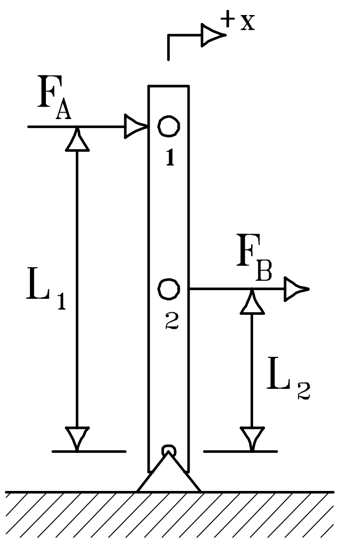
$$T_A \left(-\frac{N_2}{N_1}\right)\Omega_{2g} = -T_B \Omega_{2g}$$

$$T_A = \left(\frac{N_1}{N_2}\right) T_B$$

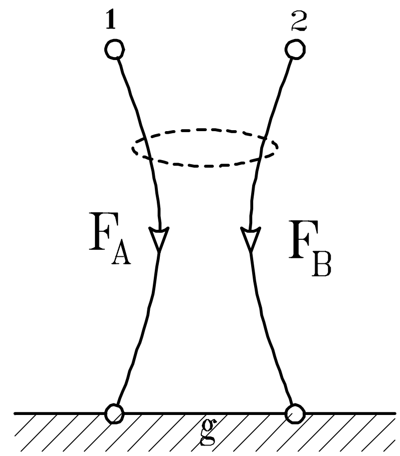
Linear Graph Symbol And Sign Conventions.

The linear graph symbol for transformer and transducers consists of two branches, one for each side of the interface. The through variables on each side of the interface must be named, such as F_A and F_B, so that they can be used in equations. The transformer or transducer is identified by one of the equations written above the symbol.

$$V_{1g} = \left(\frac{L_1}{L_2}\right) V_{2g}$$



Translational Mechanical Transformer



Linear Graph Transform Symbol

A dashed ellipse encircling the two branches of the interface indicates that there is a power flow across interface. **Note: The through variables do not flow across the interface!** There is not branch connecting nodes 1 and 2. This is (or should be) obvious in a transducer because the through variables on either side of the interface are different. In an electric motor, they are current and torque. There can not be a branch connecting the nodes across the interface in a transducer because the continuity equation would not make sense. For example, you cannot add currents and torques at a node. They are different physical quantities. Although it is less obvious in a transformer because the subsystems on either side have the same units, a branch across the interface is just as meaningless. F_A and F_B are different forces because they act at different points on the lever. Their interrelationship is due to moment equilibrium about the pivot, not a force balance which includes a mystery force flowing between nodes 1 and 2. There is no branch connecting nodes 1 and 2.

In ME 352, transformer and transducer branches will be referenced to ground, but there are many situations in machine design in which the pivots of levers and the bearings of gear shafts are not fixed to ground. In ME 352, to keep our signs correct, **the flow of the through variable in both branches of the transformer or transducer is oriented towards ground**. (Recall that our sign conventions also require the through variables in sources to flow away from ground. Sources, transformers, and transducers are the only elements in which our sign convention dictates the assumed flow direction.) Remember, the through variable flow by itself does not represent the power flow across the transducer! Power is the product of the through variable and the across variable!

The sign convention for mechanical transformers and transducers requires us to:

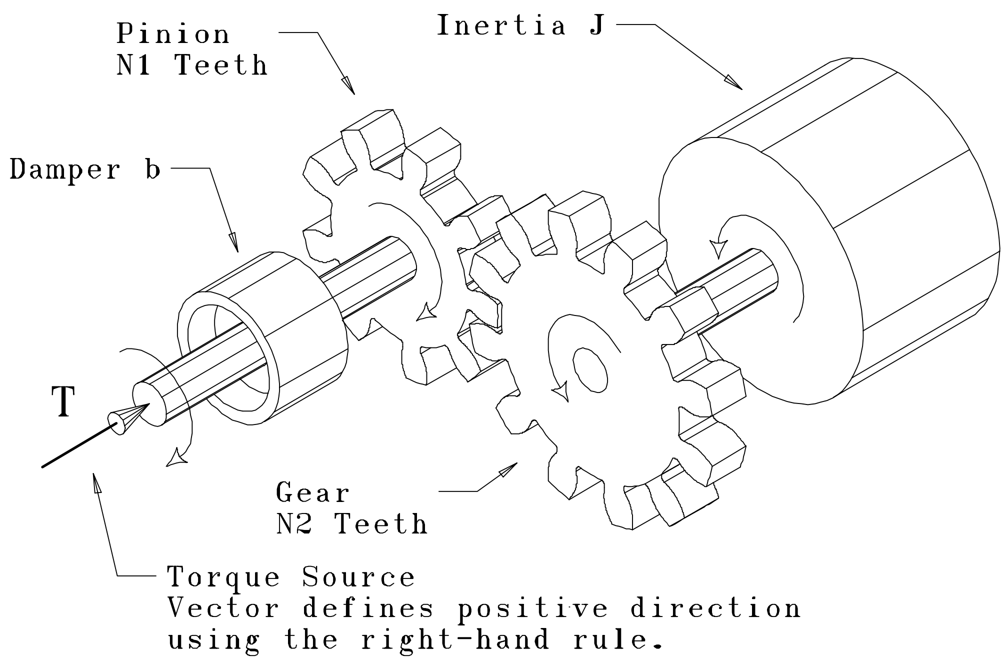
1. Assume both power flows are into the transformer or transducer, $\mathbf{P}_A = -\mathbf{P}_B$.

2. Show the forces or torques acting on the transducer or transformer in the positive direction.

3. Orient the through variable flow on both branches of the linear graph symbol towards ground.

Example: The Linear Graph for a System with a Transformer.

Following these three conventions will take care of the signs and eliminate errors due to interchanging action and reaction at the transducer or transformer. This is illustrated in the rotational mechanism below which consists of a torque source, rotational damper, gear set, and rotational inertia. Assume that the shafts are very stiff, have negligible inertia, and, consequently, can be modeled as having no energetic properties.

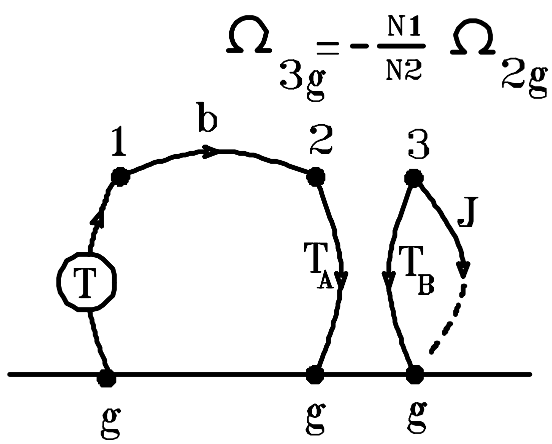


Draw the linear graph by first finding the nodes of distinct values of the across variable angular velocity Ω. The torque source must react against ground. The two shafts attached to the rotational damper have different velocities. The gears are counter-rotating and will have different angular velocities even if they have the same number of teeth. The inertia is rigidly attached to the gear and has the same velocity. Hence, there are three distinct angular velocities plus ground. The transformer relationship we derived above will be used here, revising the subscripts for the angular velocities to be Ω_{2g} for the pinion and Ω_{3g} for the gear:

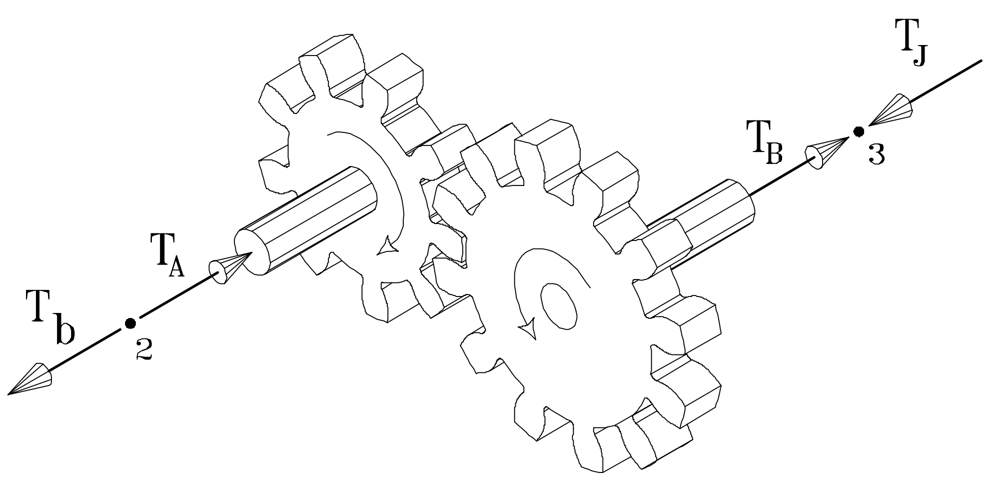
$$\Omega_{3g} = -\left(\frac{N_1}{N_2}\right)\Omega_{2g} \quad \text{and} \quad T_B = \left(\frac{N_2}{N_1}\right)T_A$$

Both of these equations are added to our equation list. One of these equations must be written above the transformer symbol on the linear graph to identify it.

We need to define the positive direction for torques and angular velocities. If a positive direction is not indicated explicitly then use the direction of the source. Because the gear set is an interface between two subsystems, it has two branches in its linear graph symbol. **The torques shown on the branches of the linear graph symbol for the transducer are THE TORQUES ACTING ON THE GEARS, which ARE ASSUMED TO POSITIVE TOWARDS GROUND.** Our linear graph is:

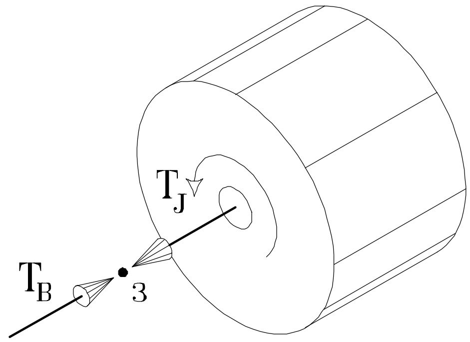


Now, lets confirm that our sign conventions lead to signs for the transformer that make sense. The torques T_A and T_B shown on the linear graph are positive torques **ACTING ON** the gear set, as shown below:

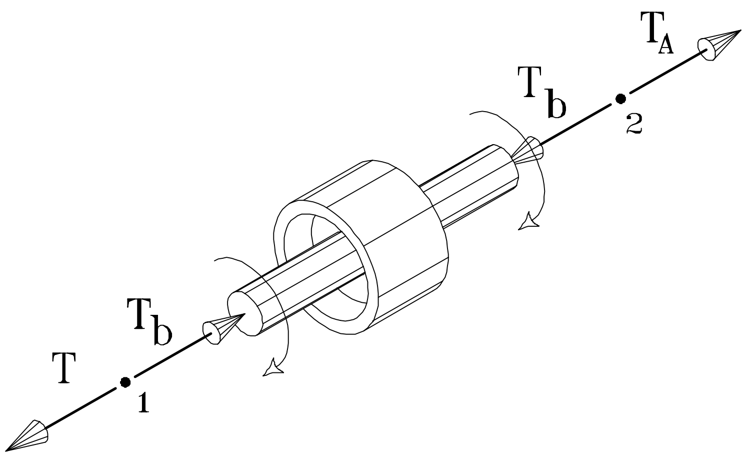


The reaction torque at node 2 from the damper against the pinion, T_b, and the reaction torque at node 3 from the rotational inertia against the gear, T_J, are also shown. The torques T_b and T_J shown are the torques **ACTING ON** the damper and the rotational inertia, respectively.

These are the torques whose signs must make sense physically. Starting with the inertia, it will rotate in the same direction are the gear, counter-clockwise for the given input torque. Using the right-hand rule on the vector T_J, we see that it produces counter-clockwise rotation of the inertia.



Considering the rotational damper, we must have equal and opposite torques acting on the two shafts of the damper. Torque T_A at node 2 is the torque acting on the pinion. It is in the positive direction. The reaction torque is the torque T_b, acting in the negative direction. Similarly, at node 1, torque T_b is the torque acting on the damper. The reaction torque T is the torque of the damper on the torque source.

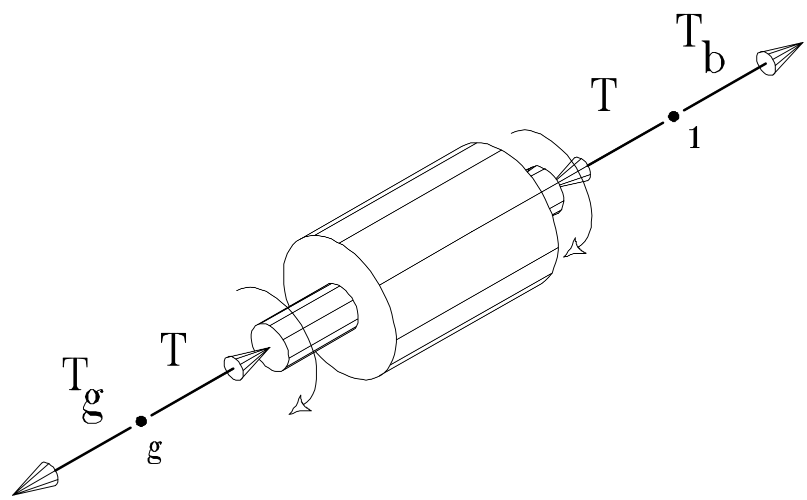


There are two aspects of this drawing that look wrong. First, why is the angular velocity of node 2 is in the opposite direction of torque T_b? Because in order for torque T_b to act through the damper, the damper must be in quasi-static equilibrium. (The prefix "quasi" means "as if" or "in a sense". In this context, it means we can formulate torque equilibrium even though the damping is in motion.) The torques acting on the damper must sum to zero. The elemental equation for the damper:

$$T_b = b\Omega_{12} = b\left(\Omega_1 - \Omega_2\right)$$

states that there needs to be a difference in angular velocity between the two ends of the damper for a torque to exist. The two ends need not move in opposite directions. In fact, they generally move in the same direction

Second, if we have assumed that the applied torque T acts away from ground, why is torque T at node 1 in the acting towards ground? This is also a case of action and reaction. If we were to draw a free body diagram of the device serving as our torque source, we would see that for the torque source to be in quasi-static equilibrium and for torque T to act away from ground at the ground (node g), then the torque T acting at the other end of the torque source at node 1 must be in the opposite direction.



In summary, the network (linear graph) representation and its sign conventions allow us to avoid necessity of drawing free body diagrams for every mechanical element in a system, but it is important to follow the conventions.

Transducer Equations

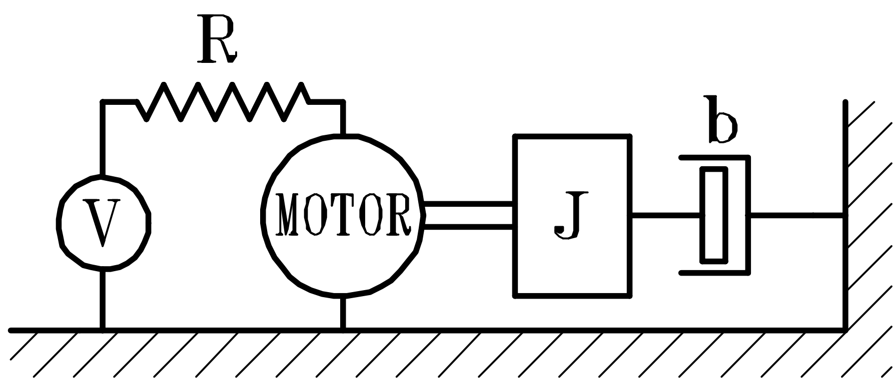
Fluid systems are mechanical systems in that they obey Newton's laws, but there is a sign reversal between solid mechanics and fluid mechanics. Positive pressure occurs when a fluid is compressed; negative (absolute) pressure when the fluid is in tension. When working with fluid system transducers, such as a piston-cylinder, define your sign convention such that a compressive force transmitted to a fluid produces positive pressure, and then use $\mathbf{P}_A + \mathbf{P}_B = 0$ to derive the relationship between displacement and volume flow rate. This sign convention avoids negative fluid pressure. Negative pressure signals possible fluid cavitation (vaporization), which is very destructive to machines. The bubbles caused by the vaporization are not the problem, although in general you want to avoid "multiphase" flow, the damage caused by cavitation is created by the high stresses which occur when the bubble collapses.

It is often not possible to derive a relationship from the information provided about a system, either because the information is insufficient or an analysis of the phenomenon is too complex to perform with confidence, or both. In those cases, experimental measurement must be performed to establish a relationship between the a power variable on either side of the interface. A case in point are the measurements you made in ME 332 to determine the relationship between the motor current and torque of the DC motor you used in your ME 210 dragster.

The sign conventions for electromechanical transducers (motors) are arbitrary. The motion of a motor in response to an applied voltage is consistent relative to the housing of the motor but independent of the orientation of the motor in space. Pick a motor up and turn it around; it will rotate in the opposite direction relative to a fixed reference frame. Some motors are "dual shafted" (which is really a single shaft that extends out the "front" and the "back") to further complicate signs. Be very wary when designing with motors because you cannot be sure of the positive direction of rotation until you test it. If you have to guess, guess the "right-hand rule" with your thumb pointing away from the motor along the axis of the shaft. The sign convention for electrical transformer is indicated on the electrical schematic symbol for a transformer as a dot on a lead to each of the coils representing positive.

Transducer Equations for a Permanent Magnet DC Motor.

DC motors interface electrical and rotational mechanical subsystems. The schematic for a DC motor model, neglecting the induction of the windings, is shown in the schematic below.



Other than the voltage source, all of the elements shown in this schematic are energetic properties of the DC motor. The motor windings have contact and conductor resistance which are combined as R. The rotating element in the motor, called the armature, are made of "electrical steel" (formulated for a high magnetic permeability to enhance the magnetic field created by the current through the windings) and copper wire windings. The kinetic energy storage of these components is combined in inertia J. There is frictional energy loss in the bushings or bearings which support the shaft. This is modeled as rotational damping b. The transducer property of the DC motor is indicated by the circle labeled "motor". This transducer interfaces the electrical and mechanical subsystems. It is important to emphasize that the transducer property neither stores nor dissipates energy. Any energy storage or dissipation in a real transducer must be represented by elements in addition to the transducer element. The transduction can be represented using the transformer block, with both power flows flowing in, as shown below, although one flow must be negative. In the linear graph symbol, both through variables, current i_M and torque T_M , flow to ground. Both across variables, voltage v_{2g} and angular velocity Ω_{3g}, are referenced to ground.

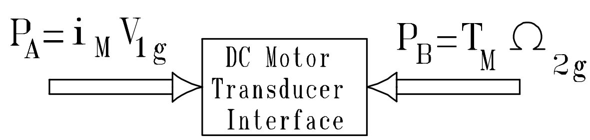
The product of the current i_M and voltage between node 2 and ground on the electrical side of the interface is the electrical power which flows into the transducer:

$$\mathbf{P}_{\text{Electrical}} = i_M v_{2g}$$

Likewise, the product of torque T_M and angular velocity between node 3 and ground on the electrical side of the interface is the mechanical power which flows into the transducer:

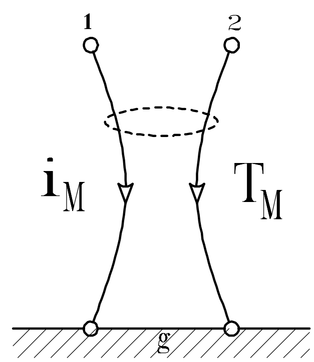
$$\mathbf{P}_{\text{Mechanical}} = T_M \Omega_{3g}$$

We must experimentally determine (or be given) one of the two transducer equations. Assume that we experimentally determined $T_M = K_T i_M$.



Block Diagram of Power Flows

$T_M = K_T i_M$



Linear Graph Symbol

Given the experimentally determined the relationship:

$$T_M = K_T i_M$$

and knowing:

$$\mathbf{P}_A = -\mathbf{P}_B$$

we can derive the relationship between the voltage drop across the transducer, v_{2g}, and the angular velocity, Ω_{3g}.

$$i_M v_{2g} = -T_M \Omega_{3g}$$

$$i_M v_{2g} = -K_T i_M \Omega_{3g}$$

$$v_{2g} = -K_T\Omega_{3g}$$

Checking the units of a system equation is more difficult if it there is a transducer in the system because the transducer constant appears to have different units in each of the transducer equations. In fact, the two sets of units are the same, if expressed in fundamental units. However, do not check the units of the system equation by expressing the transducer constant in fundamental units. It is easier to simply check the system equation units by expressing the transducer constant units in terms of both sets of power variable units and then making the substitution for the transducer constant units last, when it will be clear which set to use, rather than by expressing all of the parameters of the system in fundamental units, which often produces more errors than it reveals.

Example: Express the units of the motor torque constant K_T in terms of the power variables.

$$[T] = [K_T i]$$

$$\therefore \quad [K_T] = \left[\frac{T}{i}\right]$$

and

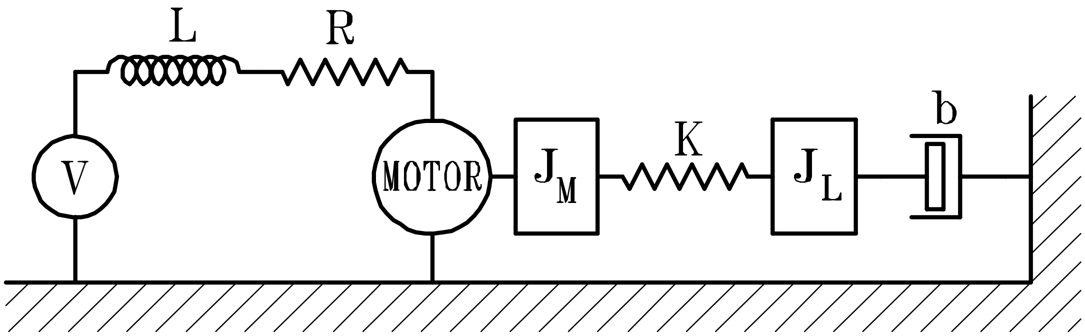
$$[v] = [K\Omega]$$

$$\therefore \quad [K_T] = \left[\frac{v}{\Omega}\right]$$

The terms “gyrating” and “non-gyrating” transformers or transducers are definitions that academics use that have no practical significance. A “non-gyrating” transformer or transducer is one whose power equations relate the same type variables to one another. The DC motor is “non-gyrating” because it relates torque to current, and these are both through variables. A gyrating transducer relates a through variable in one subsystem to an across variable in the other.

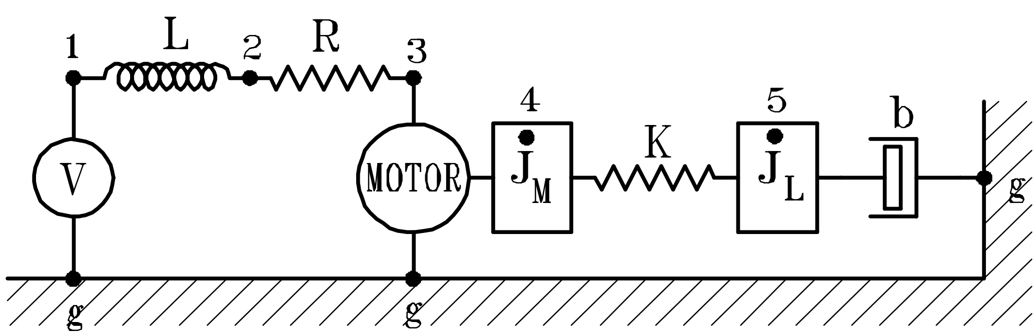
Example: The Linear Graph for a System with a Transducer.

Consider a dynamic system with a permanent magnet DC motor driven by a voltage source. The motor has significant electrical induction and resistance and rotational inertia. The motor drives a inertial load attached to it by a shaft with appreciable compliance. The inertial load rotates on hydrodynamic bearings with viscous friction. A schematic of this system is shown below.

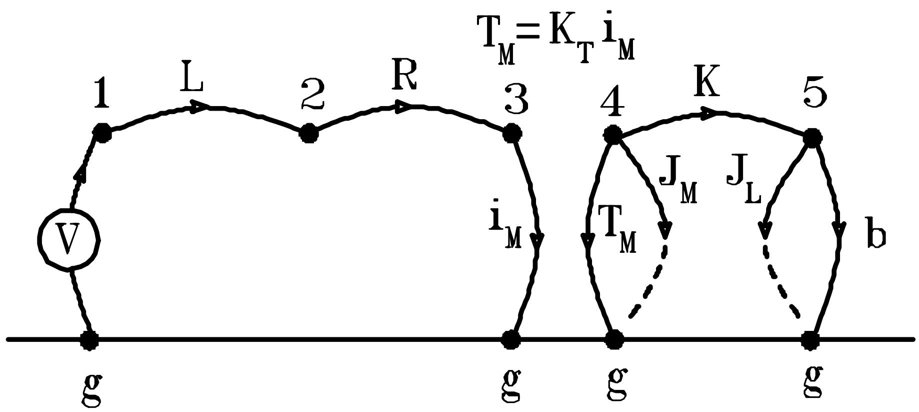


Note that there is a dynamic element in the schematic for each energetic property of both the motor and the load. The circle labeled "Motor" represents the transducer property of the motor.

The first step in drawing the linear graph for this system is to **FIND THE NODES OF DISTINCT VALUES OF THE ACROSS VARIABLES**. What are the across variables of these two systems? The across variable of the electrical system is voltage. The across variable of the rotational mechanical system is angular velocity.



Draw the linear graph by drawing and identifying the nodes and then adding the elements between them. Remember that the transducer property of the motor interfaces the electrical and mechanical subsystems. The interface is between the electrical resistor R and the rotational inertia J_M.



One would produce the equation list for this system as usual. This system has four independent energy storage elements, the inductor, and two rotational inertias separated by the rotational spring. We will not reduce a system as complex as this to a single differential equation. We will perform a partial reduction to a set of four first order differential equations using the "state-space" method.

The first two problems ask you to find an "equivalent" element to replace a mechanical transformer (gear or lever) and an energy storage or dissipation element. What does "equivalent" mean in this context? You are dealing with energetic systems. Therefore, the relevant "equivalent" behavior is equivalent energy storage or dissipation. These problems are asking you to size an element that will store (or dissipate) the same amount of energy (or power) as the mechanical transformer plus element it replaces. To determine the equivalent element, draw the linear graphs of the original and the equivalent systems and write the equation list for each. Note that the elemental parameter of the original and the equivalent elements appears in the elemental equation, which contains two power variables, and, if it is an energy storage element, in the energy equation, which contains a power variable. The equivalent elemental parameter is derived by starting with the elemental or energy equation of the original system and making substitutions to eliminate the power variable of the original equation and replace them with the power variables of the equivalent equation. This process will introduce a constant term with the transducer or transformer constant. When the result is rearranged so that it is in the form of the original equation, the original elemental parameter will be multiplied by the constant term. This product is the equivalent elemental parameter.

You will need to use geometric comparability, equilibrium, and energy conservation (written in terms of power) to understand levers and gears. When working with levers, remember that they are rigid and assume that they are going through small displacements so you can represent the motion of a point on the lever as a straight line rather than an arc.

Problem Set No. 5

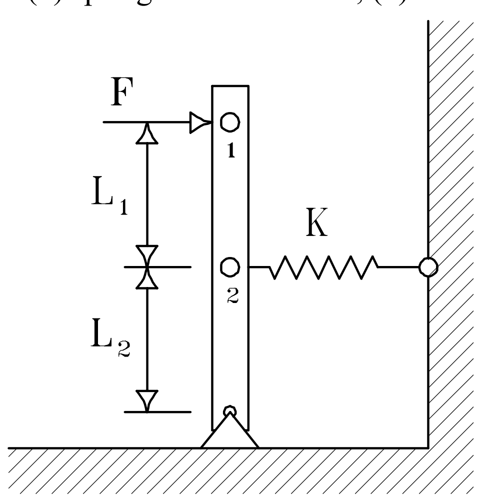
Due Friday, March 2, 2001

Read: Rowell and Wormley, pp.169–187, study the figures. Read pp. 120-124.

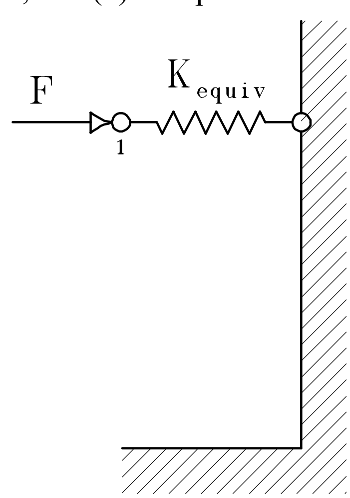
Problem 1: A translational mechanical system, the "Original System", is shown in the figure below. A force F acts at the top of a lever. A mechanical element, a spring, is attached between the lever and ground. It is possible to simplify this design and replace the subsystem consisting of the lever and the spring with a single "equivalent" spring. The equivalent spring must have the same relationship between the applied force F and the velocity of the node at which it is applied as did the original system when the force was applied to the top node of the lever for the two systems to be dynamically "equivalent". In other words, if the force F were applied by an operator then both systems must have the same "feel". If the element is an energy storage element, as is the spring, then the equivalent element must also store as much energy as the original spring when acted on by the same applied force. These two criteria lead to the same result and either can be used to derive the equivalent element.

Proceed as follows: Draw the linear graph for the original system, derive the transformer equations, then list the continuity, compatibility, elemental, and energy equations for the original system. Draw the linear graph for the equivalent system and list the continuity, compatibility, elemental, and energy equations for the equivalent system. Note the power variables used in equivalent system. Using your equation list for the original system, start with either the elemental or energy equation for the original element and then, by substitution, eliminate the power variables for the original element and replace them with power variables for the equivalent element. There will be a constant term which includes the original elemental parameter and the lengths used in the transformer equation for the lever. Rearrange the equation so that this constant term is in the same position as the parameter term in the elemental or energy equation you started with. The constant term is your equivalent element.

Determine the equivalent elemental parameter for three cases; when the mechanical element is a (1) spring with stiffness k, (2) mass with mass M, and (3) dashpot with damping b.



Original System



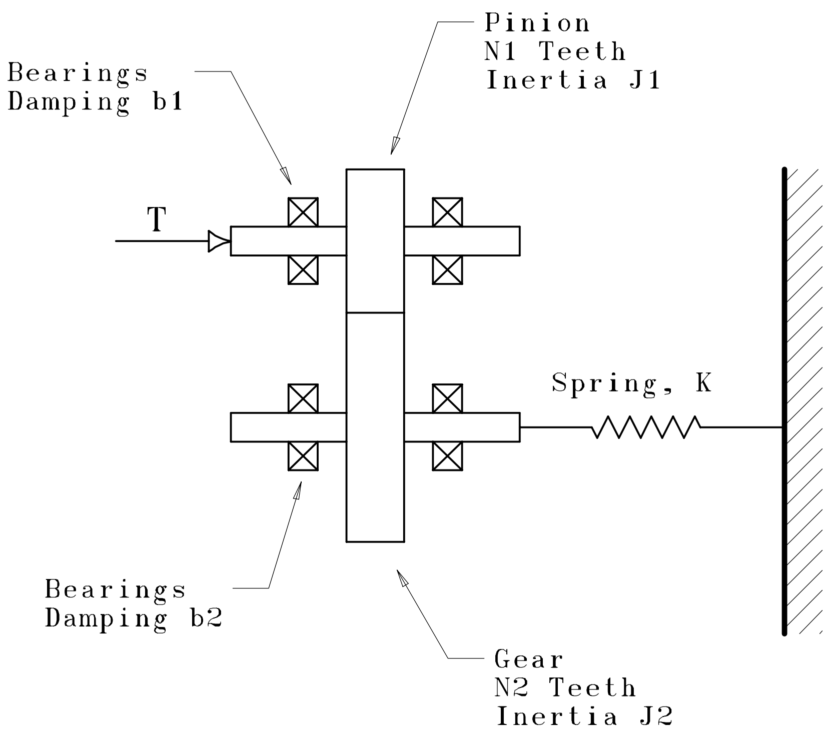
Equivalent System

Problem 2: A rotational mechanical system is shown in the figure below. A torque T acts on the input shaft of a gear set. A rotational spring with spring constant K is attached between the output shaft and ground. The pinion has N1 teeth and inertia J1. The gear has N2 teeth and inertia J2. The bearings on the input shaft collectively have damping b1. The bearings on the output shaft collectively have damping b2.

(a) Draw the linear graph of the existing system and determine the transformer equations.

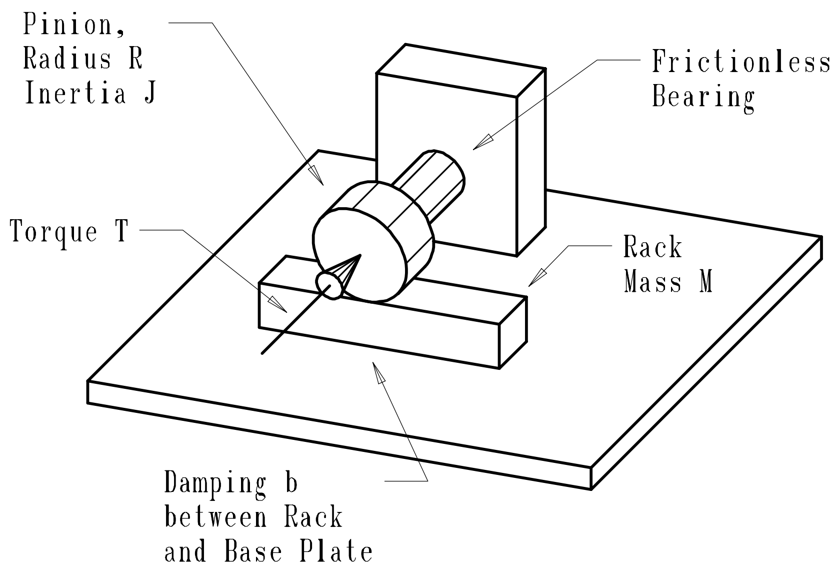
(b) Draw the equivalent linear graph if the transformer is eliminated and inertia J2, the bearings b2, and the rotational spring are replaced by equivalent elements attached to the input shaft.

(c) Calculate the equivalent elemental parameters for the equivalent system of part b.

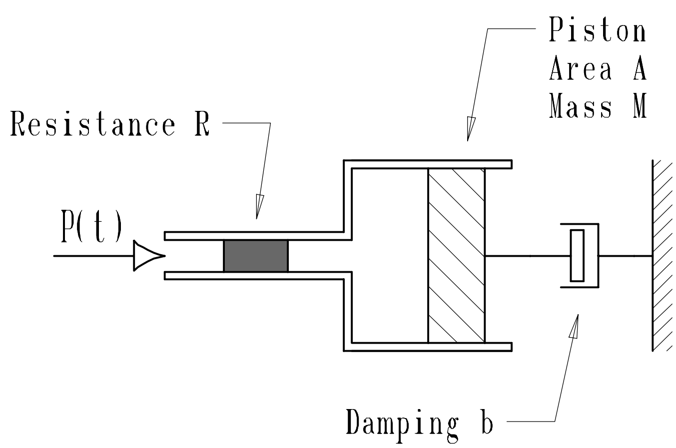


Problem 3: A mechanical system consisting of a torque source and a rack and pinion is show in the schematic below. The torque source acts on the pinion which is mounted in an ideal frictionless bearing. The pinion with inertia J =0.05 Kg - m^2 and pitch radius R = 2 cm is engaged in a rack with mass M = 150 Kg which slides along the base plate. There is viscous friction between the rack and the base plate which can be modeled as linear damping, with damping coefficient b = 20 N - sec / m.

Derive the system equation which relates the applied torque to the velocity of the rack and check its units. The system is a rest at time t = 0^- when a torque T = 10 N-m is applied. Solve the system equation. Plot the response using Mathematica or MathCAD.



Problem 4: Draw the linear graph for the fluid mechanical - translational mechanical system shown below. Derive the system equation that relates the applied pressure to velocity of the piston. The system is de-energized for time t < 0. A step input in pressure P is applied at time t = 0. Use Mathematica or MathCAD to plot the response of this system given P = 10, R = 3, A = 5, M = 2, and b = 4.



Problem 5: A DC motor which drives a rotational inertia connected to a damper, as shown in the schematic below. The relationship between motor current and torque is $T = K_T\ i$, where K_T is 20 N-m/amp. The resistance $R = 10\ \Omega$, $J = 1$ Kg-m^2 and $b = 0.5$ N-m-sec. The compliance and inertia of the shaft are negligible. The system is de-energized for time $t < 0$. Derive the differential equation that relates the angular velocity of the rotational inertia J to the applied voltage. Use Mathematica or MathCAD to plot the response of the motor to a 24 V step input.