

# Annotated Bibliography on Fair Division and Social Justice

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## References

- [1] I. Bezakova and V. Dani. Nobody left behind: Fair allocation of indivisible goods. 2006.

*Abstract:* The Max-Min Fairness problem is as follows: Given  $m$  indivisible goods and  $k$  players, each with a specified valuation function on the subsets of the goods, how should the goods be split between the players so as to maximize the minimum valuation. Viewing the problem from a game theoretic perspective, we show that for two players and additive valuations the expected minimum of the (randomized) cut-and-choose mechanism is a  $1/2$ -approximation of the optimum. To complement this result we show that no truthful mechanism can compute the exact optimum. We also consider the algorithmic perspective when the (true) additive valuation functions are part of the input. We present a simple  $1/(m - k + 1)$  approximation algorithm which allocates to every player at least  $1/k$  fraction of the value of all but the  $k - 1$  heaviest items. We also give an algorithm with additive error against the fractional optimum bounded by the value of the largest item. The two approximation algorithms are incomparable in the sense that there exist instances when one outperforms the other. We conclude with a  $1/2 + \varepsilon$  factor NP-hardness of approximation result.

*Annotation:* This article has a great title, and the method developed seems both interesting and applicable. Note that the valuation function allowed in this paper permits synergistic evaluations, *e.g.*,

the value of a pile containing both a boat and its motor might be substantially more than the sum of the values of the two separately. (This example arises but is not addressed in chapters 4 and 5 of [5].) There are no explicit associations with issues of social justice in this article, however. It seems best suited to an advanced mathematics audience, as its content is quite sophisticated; I didn't try to read the exposition.

- [2] S. J. Brams and T. R. Kaplan. Dividing the indivisible. *Journal of Theoretical Politics*, 16(2):143–173, 2004.

*Abstract:* Political parties in Northern Ireland recently used a divisor method of apportionment to choose, in sequence, ten cabinet ministries. If the parties have complete information about each others preferences, we show that it may not be rational for them to act sincerely by choosing their most-preferred ministry that is available. One consequence of acting sophisticatedly is that the resulting allocation may not be Pareto-optimal, making all the parties worse off. Another is non-monotonicity choosing earlier may hurt rather than help a party. We introduce a mechanism, combining sequential choices with a structured form of trading, that results in sincere choices for two parties that avoids these problems. Although there are difficulties in extending this mechanism to more than two parties, other approaches are explored, such as permitting parties to make consecutive choices not prescribed by an apportionment method. But certain problems, such as eliminating envy, remain.

*Annotation:* This article is a theoretical application of fair division ideas to political power-sharing, and specifically to appointing cabinet ministers in Northern Ireland. I have only read the abstract, but based on timing and topic, I believe that the article applies the Gap Procedure developed in [3] to the problem at hand.

- [3] S. J. Brams and D. M. Kilgour. Competitive fair division. *Journal of Political Economy*, 109(2):418–443, 2001.

*Abstract:* Several indivisible goods are to be divided among two or more players, whose bids for the goods determine their prices. An equitable assignment of the goods at competitive prices is given by a fair-division procedure, called the Gap Procedure, that ensures (1) nonnegative prices that never exceed the bid of the player receiving

the good; (2) Pareto optimality, though coupled with possible envy; (3) monotonicity, such that higher bids never hurt in obtaining a good; (4) sincere bids that preclude negative utility; and (5) prices that are partially independent of the amounts bid (as in a Vickrey auction). A variety of applications are discussed.

*Annotation:* The canonical example in this paper is assigning rooms to housemates based on the prices that individuals are willing to pay for the privilege of occupying a room. The Social justice aspect of this article is only tangential. In the final paragraph, the authors point, “Other political applications of the Gap Procedure come to mind in which players do not have equal endowments or entitlements, as in bidding for ministerial posts in a parliamentary government.” This idea is developed more fully in [2].

- [4] S. J. Brams and D. L. King. Efficient fair division: Help the worst off or avoid envy? *Rationality and Society*, 17(4):387–421, 2005.

*Abstract:* Two or more players rank a set of indivisible items from best to worst. An efficient allocation of items is characterized, which may satisfy such properties as maximin, Borda maximin, and envy-avoidance. Whereas the two maximin properties are in conflict with envy-avoidance, there is always an efficient allocation that does not ensure envy, but it may not be maximin or Borda maximin. Computer calculations show that maximin allocations lead to envy quite often, but Borda maximin allocations do so only rarely. Implications of the theoretical findings for real-world fair-division problems are discussed.

*Annotation:* The title of this article suggests social justice considerations, but the connections are rather weak.

- [5] S. J. Brams and A. D. Taylor. *The Win-Win Solution: Guaranteeing Fair Shares to Everybody*. W. W. Norton & Company, 1999.

*Annotation:* This is my intended text for the fair division module of my course. It is the least technical presentation of the material, and actually avoids mathematical exposition entirely. This is appropriate for the course I am teaching, but may well not be for a course that in mathematics. For such a course I would recommend [6].

In general, the issue is the division of a collection, or pile, of goods amongst interested parties, who are all assumed to be worthy of equal treatment. The book discusses and elaborates on three basic division

procedures: alternation, divide and choose, and adjusted winner. In alternation, the parties take turns choosing parts of the pile for their portion. In divide and choose, one party divides the pile and allows the other(s) to choose the portion they want, leaving the divider with the remaining unclaimed portion. Adjusted winner assigns each item in the pile to the party who values it most, and then adjusts the assignment to be equitable, as defined below.

The primary goals for the procedures covered is to make the division (and assignment) envy-free, that is, each party believes that their portion is at least as desirable as any other. This also offers the possibility of *each* party receiving a part that it perceives as “generous,” that is, the party values the assigned portion more than  $1/n$  of the value of the entire pile, where  $n$  is the number of parties.

When different items of the pile are assigned different values by different parties, the concept of Pareto-optimality comes into play, and here it is given the name efficiency. That is, we would like to maximize the total value that the parties assign to the pieces they receive. As one might expect, the constraint of envy-freeness might prevent the achievement of the Pareto-optimal solution, but it is possible to optimize the total value within the constraint of envy-freeness.

A refinement of envy-freeness is an equitable distribution. The idea is to make each party’s perceived value assignment equal. The adjusted winner process results in a division between two parties that is envy-free, and equitable. It is also efficient within the constraint of equitability.

- [6] Steven J. Brams and Alan D. Taylor. *Fair Division: From Cake-cutting to Dispute Resolution*. Cambridge University Press, Cambridge, England, 1996.

*Annotation:* This has to be the most important entry in this bibliography. the text presents the mathematics of fair division, evidently with a modicum of rigor. When I read it, I’ll be able to say more.

- [7] D. B. H. denoon and S. J. Brams. Fair division: A new approach to the Spratly Islands controversy. *International Negotiation*, 2(2):303–329, 1997.

*Abstract:* The Spratly Islands are a group of over 230 small islands and reefs in the South China Sea. Both China and Taiwan, as well as four members of the Association of Southeast Asian Nations (ASEAN) Vietnam, the Philippines, Malaysia, and Brunei have made claims

on part or all of the land areas and surrounding waters, which are believed to have major oil and gas deposits. Because there are overlapping claims, and no single country has had continuous possession of the area, it is unlikely that international legal procedures can resolve the dispute quickly. There are four major issues in the dispute: sovereignty, economic development, freedom of passage, and regional security. We focus on sovereignty and suggest a two-step process for dividing the islands. A fair-division procedure called Adjusted Winner (AW) would first be applied to the allocation between China and ASEAN, after which there would be an allocation among the ASEAN states. The example used to illustrate AW divides the region into five zones and concentrates on the first step, negotiation between China and ASEAN. We present three potential bidding strategies for China, and two for ASEAN, that give six different allocations of the islands. The AW allocations are efficient, equitable, envy-free and, in our example, give both sides between 65% and 83% of what we assume they would prefer.

*Annotation:* This is another application of the theoretical work of Brams and Taylor to a real-world problem with social justice implications, but as in [2], it is not being applied by the interested parties, only by the authors in a conjecture.

- [8] Y. Dutil and Y. Dutil. Social choice and equity theories: Seeking the common good as a common ground. *55 th International Astronautical Congress 2004*, pages 1–8, 2004.

*Abstract:* The communication of basic mathematical and physical notions is generally accepted, as one of the very first concepts that will be exchange between civilisation, since math and physics appears universal. However, such concepts only reflect a small fraction of a much larger body of knowledge to might be share between civilizations. Notoriously, no obvious scheme for exchanging knowledge as been proposed for social sciences, which unlike physical science, are not mathematically formalised. However, some branches of social science have been mathematically formalised. Social choice theory, which deals with the seemingly simple but fundamentally complex problem of finding the optimal "common good" from individual choices, offers a direct path to discuss to notion of democracy. Equity theory, which treats the complex problem of sharing goods or chores, is a very pow-

erful tool to explain the notion of distributive justice. Since democracy and justice provide many societal advantages, such notions are probably universal. Due to the importance of those tools for the survival of civilisation, they are probably among the most useful information to share with another civilisation.

*Annotation:* Sadly, the title seems to be the most useful part of this article. that abstract seems turgid and vague, and the Proceedings in which it is published will be very difficult to obtain.

- [9] H. Gersbach. Dividing resources by flexible majority rules. *Social Choice and Welfare*, 23(2):295–308, 2004.

*Abstract:* We examine the division of resources among individuals by flexible majority rules where the majority necessary to adopt a proposal depends on the proposal itself. For instance, the size of the majority may increase with the maximal difference between the shares individuals receive. For large discount factors such rules imply an efficient and even distribution of resources. For low discount factors flexible majority rules supplemented by specific agenda-planning rules such as agenda rights for the opposition guarantee envy-free distribution. Uncertainty about discount rates can make it easier to achieve efficient and envy-free allocations.

- [10] J. M. Greig. *Breaks in the Storm: Recognizing ripe conditions for mediation between disputing states*. PhD thesis, University of Illinois, Champaign-Urbana, 2002.

*Abstract:* This study examines the contextual conditions associated with both the initiation and success of international mediation efforts between disputing states. Most research on mediation success has tended to focus upon short-term outcomes such as the establishment of a mediation agreement or a cease-fire. Little attention, however, has been devoted to the contextual factors that promote broader improvement in the relations between states beyond the simple achievement of an agreement. Because mediation agreements do not necessarily translate into significant improvement in the relationship between disputants, this lack of attention to the broader effect of mediation represents a significant shortcoming within the mediation literature. The theoretical model developed in this study distinguishes between the contextual factors associated with mediation success across two

distinct dimensions. First, the model distinguishes between the contextual factors associated with mediation agreements and those associated with an actual reduction in conflict between states. Second, the model distinguishes between the contextual factors favorable for mediation success based upon the stage of the conflict between the disputants. By focusing upon both the initiation and success of mediation, the study permits a comparison of the factors that stimulate mediation efforts to those that promote their success. The results of this study permit the reconciliation of some of the divergent theoretical expectations within the mediation ripeness literature as well as the development of a better understanding of when international mediation is most likely to be successful.

*Annotation:* This thesis has clear social justice implications, but its connections to fair division are more tenuous.

- [11] J. M. Greig and P. F. Diehl. The peacekeeping—peacemaking dilemma. *International Studies Quarterly*, 49(4):621–646, 2005.

*Abstract:* Peacekeeping has become an increasingly prominent tool for conflict management and there has been an accompanying explosion of scholarly studies on peacekeeping. Yet, such analyses typically ignore the process of getting a peace agreement itself, missing the potential impact that a peacekeeping force might have in facilitating a peace agreement between protagonists. In this paper, we explore among both enduring rivalries and civil wars whether the presence of a peacekeeping force enhances the prospects for gaining an agreement between protagonists. The academic literature suggests opposing logics: one suggesting the desirability of peacekeeping forces while the other implies that they may be counterproductive. We consider whether the presence of peacekeeping enhances or inhibits mediation and negotiation attempts. We also explore whether the success rates for international mediation and negotiation efforts in those conflicts are affected by the presence of peacekeeping forces. Our results suggest support for the pessimistic view of peacekeeping as it discourages diplomatic efforts and decreases the likelihood of achieving a settlement, although the results are clearer for interstate conflict than for civil wars.

- [12] A. K. Haugestad. The dugnad: Sustainable development and sustainable consumption in Norway. *Presentation at the 6th Nordic Conference on Environmental Social Sciences (NESS), June, 2003.*

*Abstract:* Debates on sustainable development and ecological modernization have drawn attention to potentials for win-win solutions in the production of goods and services. The paper explores potentials for win-win solutions at the consumption level. This is done through an investigation of potentials for changing norms for being a “good” (successful, responsible) consumer in Norway. The potential is presented as an alteration from getting as much comfort, experiences, goods and services as possible out of ones purchasing power (a narrow household perspective) towards choices that balance the narrow household perspective with a concern not to use ones purchasing power at the expense of the environment or other peoples welfare (global responsiveness). In the paper “limits to a guilty conscience” is linked to the Norwegian “dugnad tradition.” According to this tradition everybody is supposed to contribute with his or her time and work to the common good. From a dugnad perspective the global struggle for sustainable development is a global dugnad. The discussion of the dugnad culture builds on literature on Norwegian history, participant observation of Norwegian culture and qualitative interviews with 28 Norwegians from the whole range of political parties. The interviews dealt with attitudes to consumption and distribution in todays world and the most important finding was the discovery of *Homo politicus norvegicus*—the ideal typical dugnad leader. The paper also raises the question whether an apparent potential for changing norms towards globally responsible consumption is due to specific traits in Norwegian political culture or whether similar discursive resources might be expected to be readily available within other political cultures as well. There seem to be reasons to assume that a global resource sharing perspective on sustainable development might release win-win possibilities for rich consumers all over the world. They would have to give up some consumption privileges, but get a better earth citizen conscience in return. Another suggested reason for assuming that a sustainable world society is politically possible, is that rich consumers and voters eventually have self-interest in a stable and peaceful world society.

- [13] K. K. Petersen, J. A. Vasquez, and Y. Wang. Multiparty disputes and the probability of war, 1816–1992. *Conflict Management and Peace Science*, 21(2):85–100, 2004.

*Abstract:* Previous theory and research have suggested that multiparty disputes might be significantly more likely to escalate to war than bilateral disputes, because of the difficulty of reaching a mutually acceptable agreement as the number of parties increases. This study presents a systematic test of this hypothesis. Efforts to provide such a test have been hampered by the absence of data that distinguish the number of participants in a militarized interstate dispute prior to the outbreak of war from the number of participants after the war breaks out. We find that multiparty disputes do have an increased probability to escalate to war. In addition, we find that the issue over which the disputants contend has an important effect on the probability that the dispute will escalate to war; multiparty disputes that are over territory have a higher probability of escalating to war than multiparty disputes in general. Lastly, it is found that the effects related to the number of parties in a dispute and to whether the dispute is over territory are independent, and one does not eliminate the effect of the other. In order to contribute to future scholarship on this topic, the data for the new classification scheme of multiparty disputes are published in the appendices.

*Annotation:* Like [10] this article has important social justice implications, but the connection to fair division is more tenuous.

- [14] G. Schneider and U. S. Krämer. The limitations of fair division. *Journal of Conflict Resolution*, 48(4):506–524, 2004.

*Abstract:* Mathematical procedures that promise an envy-free, equitable, and efficient solution to distributional conflicts have received widespread attention. Two fair-division mechanisms, adjusted Knaster and proportional Knaster, which are similar to the well-known adjusted-winner procedure, are compared with the less fair divide-and-choose mechanism. Results show that participants largely prefer the adjusted-Knaster procedure to the two alternatives. Adjusted Knaster, closely followed by proportional Knaster, also promises the highest average payoff. Yet the sophisticated mechanisms cease to perform better than divide-and-choose once actors receive

the possibility to deviate from the mandatory bargaining protocols of fair-division procedures. The preference for adjusted and proportional Knaster is found to be a partial function of the participants psychological profile. The more "antisocial" a participant, the more likely this respondent is to opt for a procedure with a compensatory mechanism.

*Annotation:* The Knaster procedures are not covered in the *Win-Win Solution*, although they are presented in *Fair Division*. This article might be more useful in a more mathematically intensive course.

- [15] R. M. Siverson. A glass half-full? no, but perhaps a glass filling: The contributions of international politics research to policy. *PS: Political Science and Politics*, 33(1):59–64, 2000.

*Annotation:* Full text of this article is available through JSTOR.

The article has one short and salient passage on fair division; here is a quotation from that passage:

It would be useful in the present context to be able to report that the AW method has been used to solve a significant international dispute, but, alas, such is not the case. However, it has been used successfully in a context some may see as disputatious as an international conflict: divorce settlements (Brams 1999). While applications to international politics may as yet be absent, there are reasons to believe that AW can be useful.

Although the author goes on to cite some of the hypothetical studies mentioned elsewhere, and makes favorable mention of [7], the fact remains that these techniques are not yet used in international negotiations.