

# First-Year Seminar Course Module on Fair Division

Rob Root

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## 1 Background

This module is written as part of the course development workshop on “The Mathematics of Social Justice” held at Lafayette College in 2006. It is a draft of the module that I intend to include in my first-year seminar with the same name. The course is meant to serve a general audience of students without a strong background in mathematics, and it is not meant to substantially increase their mathematical knowledge.

Rather, the course has the following objectives:

- To introduce the students to the idea of quantitative literacy (QL), and to explore and expand their own QL.
- To apply their QL skills to the investigation and understanding of issues of social justice, specifically, the 40 year trend of increasing inequality of income and wealth in the United States will be an issue that receives sustained focus.
- To explore the attainment of QL as an issue of social justice in itself, and to investigate efforts to provide QL to students from disadvantaged backgrounds.

This module is intended to come early in the course, but after a module in which the concept of QL is developed through the reading and discussion of *Mathematics and Democracy*, and the students have the opportunity to consider their own QL in the context of the exposition in *What the Numbers Say*. More information on these books and all others mentioned here can be found on the web page at

<http://www.lafayette.edu/~math/Rob/MathOfSJ/Resources.html>.

An annotated bibliography including several sources salient to this model, and specifically the primary text, *The Win-Win Solution: Guaranteeing Fair Shares to Everybody* by Steven J. Brams and Alan D. Taylor, is available at

<http://www.lafayette.edu/~math/Rob/MathOfSJ/CourseDesc.html>.

This module is also available for download from the latter URL.

Fair division is not discussed or connected to QL in either of the books from the first module, nonetheless, I believe it makes an excellent choice for the second topic of the class

for a number of reasons. First, it is a recent development in mathematics; most of the literature on the subject has been written while these students have been alive. I hope to emphasize that mathematics is a living subject with current developments that are relevant to the students' lives, and so presenting results that have been proven in the last twenty years, and that are just beginning to make their way into common practice serves this goal admirably.

Second, the mathematics required for a basic encounter with fair division is neither utterly trivial nor especially demanding. The ability to solve linear equations in one variable is a reasonable expectation of all Lafayette students, and the appearance of such equations in a fair division process will emphasize the broad utility of basic algebra, reinforcing the connection between quantitative literacy and school mathematics.

Third, the connection between mathematics and justice in general is made stronger by considering fair division. While there are several other topics that might expand the students' QL, none can be linked so explicitly to the idea of justice as fair division. The remarkable absence of considerations of fairness in the history and theory of wealth and income inequality—the next module of the course—also serves as an interesting counterpoint, highlighting the limits of the use of fair division, and sparking reflection on the role of fairness in our society.

Finally, the ability to apply an appropriate fair division algorithm seems to be a reasonably basic and useful skill to add to the students' quantitative literacy toolbox. Knowing the divide and choose algorithm and having an appreciation for balanced as compared with strict alternation will aid students in day-to-day dispute settlement, as any parent can appreciate. Knowledge of the adjusted winner (AW) algorithm is less commonly useful, but still of considerable practical value.

## 2 Schedule of Module

The module will be relatively short, taking three 75-minute class meetings. For the first meeting, the students will be assigned the first four chapters of *The Win-Win Solution*. For the second meeting, the last five chapters of the book will be assigned. The third class will be an opportunity to discuss student (written) reflections on the import of what they learned from the text and class discussions. The class will focus on the writing process, but will naturally lead to discussions of the import of the ideas that arise in reading and earlier discussion.

### 2.1 Warm-up Activity with the first assignment

On the day the first reading assignment is made, the class will close with a five-minute cake-cutting activity. Two students will be chosen to divide a cake. The class will be asked how to divide it fairly; someone will quickly suggest cutting it in half. The class will be asked, how can we assure that the two pieces are equal in size? Some will suggest mathematical procedures (Rulers, protractors, and the like might be mentioned.) Others will question the need or efficacy for such elaborate procedures and/or devices. I hope that some student will

suggest the divide and choose algorithm. I will close by pointing out the envy-free property of divide and choose, and explain that students will learn more in the reading assigned.

## 2.2 Module Class Period 1

### Assignment:

Read the first four chapters of *The Win-Win Solution*. Pay careful attention to the exposition on the prisoner's dilemma, as this idea will appear again later in the course. Before class, try to remember a time when you were involved in a fair division, and consider the experience in light of the reading. It doesn't have to be an important event in your life; recalling the time you and a sibling or friend shared food or some object(s) would be fine. But spend some time thinking about the ways the concepts presented in the text apply (or fail to apply) to your experience. Also, consider the prisoners' dilemma as described in the excerpt called "The Story of the Prisoners' Dilemma" on page 32 of the (hardcover) book. See how many other real life situations (other than the football draft scenario presented in the book) you can think of where parties face a prisoners' dilemma.

### Discussion Questions:

I expect to spend the first half of the period discussing the first four questions, and the last half discussing the last two. If anything, I would increase the time for the last two, as the prisoners' dilemma is a concept that will come up again later in the course, in *The Winner-Take-All Society*, and I believe that understanding it and its ramifications is a useful piece of quantitative literacy, and thus is important for students in this class.

1. Describe some experiences you have had dividing things fairly. In your experience, were you treated fairly? What aspects of the experience led to a sense of fairness or unfairness?
2. Would either balanced alteration or divide and choose have improved the fairness of the outcome in your experience? What do you think would happen if you had suggested the use of one of these algorithms to resolve to division? Do you think you would be accused of applying abstract mathematics in order to confuse or dupe the party/parties?
3. If someone proposed using one of these algorithms to you, how would you respond? Do they seem fair to you? Do they seem feasible in situations you have encountered?
4. The book proposes the concepts of envy-freeness, efficiency, and equitability as the principle criteria for fairness in division. Can your sense of fairness or lack of fairness in your own experiences be expressed using these concepts? What other concepts would you use?
5. In chapters 2 and 3, the book describes the problem of sincere and sophisticated choices in terms of the prisoners' dilemma. The prisoners' dilemma is an important scenario in issues of social justice in a free market society. The free market is premised on the maximum benefit for all being attained as the result of every individual pursuing

their own interest. In the prisoners' dilemma, however, the maximum benefit for all is achieved only when individuals restrain their behavior, and use a strategy that benefits everyone rather than a strategy based on self-interest narrowly considered. The text shows that three or more parties engaged in division under either strict or balanced alteration can face a prisoner's dilemma. Can you think of other real-life situations when we face a prisoner's dilemma?

6. In the draft selection example in the book, it clearly behooves the teams to enforce sincere choices rather than sophisticated ones. How might the teams encourage sincerity and discourage sophistication? What social or regulatory conditions might improve the teams' ability to create an environment where sincerity is the dominant strategy. How might these encouragements and conditions generalize to prisoners' dilemmas in larger society?

## 2.3 Module Class Period 2

### Assignment:

Read the last five chapters of *The Win-Win Solution*. You may skip the section in Chapter 7 about Divorce if you wish. It concerns the divorces of Prince Charles and Princess Diana, and of Donald and Ivana Trump. These are not applications that are especially salient to this class, and the section does not include much exposition that is salient to uses of AW other than for divorce. The subsequent section on the Dole-Clinton presidential debates offers an adequate example for the concepts of this chapter in a situation that is at least political, if not exactly social.

### Presentation:

It may be appropriate to go over some of the mathematics in the AW procedure. Leading the class through an application of AW, possibly with a pair of students providing the valuations for a hypothetical division, might aid the class in understanding how the method works. A canned example in which the whole of one item and a portion of another are transferred from the holdings of the initial winner to the initial loser might also be appropriate; there is no example of this type in the book. It may also be desirable to go over the need to avoid valuations of zero (using a query step like that in balanced alternation) that is briefly discussed in the Spratly Islands example.

### Discussion Questions:

1. The book presents the adjusted winner strategy as optimal for two-party fair division. What difficulties do you see with implementing AW in the real-life experiences we discussed in class last time?
2. AW uses numbers explicitly in ways that the earlier strategies discussed by the authors did not. Do you think that the quantitative nature of AW would make some parties wary of its use? Do you think that the level of QL needed: the ability to solve linear equations in one variable, is one that most people are comfortable with?

3. If one party in an AW process is mathematically facile and the other is not, what advantages does that offer the former party? Is possession of QL a meaningful advantage in this situation?
4. In what ways does reducing the division to an algebraic algorithm lose human dimensions of the issue of fair division? In what ways is this reduction beneficial to the involved parties? In what ways is it deleterious?
5. The book claims that AW is efficient; your instructor claims that AW is efficient under the constraint that the outcome be equitable. What is the difference between these two statements? How, if at all, is the difference important?
6. In the last three chapters, the book applies AW to various issues of social and private justice. Do you see these issues as being important and worthy of careful consideration? Why or why not?
7. The US State Department reportedly has looked carefully at the results of the Spratly Islands analysis outlined in the text. Beyond this, however, AW has not been embraced by the diplomatic community, nor by the labor negotiation community. Why do you suppose AW has not been received more warmly? What would encourage its broader adoption? Would broader adoption result in fairer and more stable resolutions to disputes of the sort discussed in the final chapters of the book?

## 2.4 Module Class Period 3

### Assignment:

Write a reaction paper to the reading and discussion in this module. You may choose to focus on either of the questions given below, but you may write about any part of the module that attracts your interest, and you wish to consider further. Your paper should be substantial, but a classmate should be able to complete a first reading in five minutes.

1. What are the most important points made in the reading and discussion, and why?
2. Are the new techniques (balanced alternation, divide and choose, and adjusted winner) an important expansion of your QL? Why or why not?

Come to class prepared to share your paper with classmates, and to offer feedback to others. *Bring a hard copy of your paper; print it out one-sided, with one-inch margins and double spaced.* (This is so there is plenty of room for mark-up and comments from classmates.) Class will open with students exchanging papers, and reading classmates' papers. Each student will be asked to offer feedback to the paper that s/he has read. Here are the questions that your classmate will answer when reading your reaction paper, and that you will answer for the paper that you read:

- How has the paper been organized? Is there a central idea? How clearly is it stated? Does the paper as a whole build around this idea cleanly and effectively?

- Is the line of reasoning in the paper logical and clear? Are there gaps or confusing transitions?
- Are there any salient points in the module that have been omitted? Is appropriate consideration being given to the different viewpoints offered in class and in the reading? Is excessive weight being given to trivial or discredited points? Are there any irrelevant points included?
- What is the best, most original, most interesting, most important idea in the paper? Where is it located? How might the paper be reorganized to make this idea more prominent?

### **Classroom Activity**

In the class, as the assignment states, each student will read another's paper, making notes. After 15 minutes for reading, there will follow two sessions of review and critique, so that each student has the opportunity to be the reader and the writer; these will last 15 minutes each. The last half-hour of the class will be a group discussion when students can bring forward writing issues that arose during the sessions. The class may end with a warm-up activity for the wealth and income inequality module that follows.

An interesting QL sidelight: Suppose that I (the instructor) want to avoid the coziness of students having both sessions with the same fellow student, that is, I do not wish students to simply exchange papers with one another. How should papers be assigned to readers so that all students can be readers and writers in exactly two sessions? Note that this is nontrivial, since having students cycle papers in groups of three will render this impossible; one member of the group of three must be idle during each session, necessitating a third session (during which one member will be idle). It is clear that even cycles are acceptable, and every acceptable permutation of papers can be decomposed into even cycles. Further, note that the simple exchange of papers by a pair of students is possible so long as the class has an even number of students, and the decomposition of the class into even cycles, each with more than two students in it, is possible whenever the class size is even and there are more than two students.

### **Grading**

The students will revise their reaction papers and turn in both the final version and the marked-up draft. Students will be graded on the thoroughness and thoughtfulness of their critique of another's draft, and the responsiveness to the mark-up of their own drafts in their final versions.