Fluid Mechanics Review

Units

- Be consistent (SI vs. English)
- Include units through calculations, don't just add them to the answer at the end

Fluid properties

$\gamma = \rho g \qquad SG = \dfrac{\gamma}{\gamma_w} \qquad \tau = \mu \dfrac{dv}{dy}$

Statics

$$\frac{dp}{dz} = -\rho g = -\gamma$$

$\Delta p = \gamma h$ (p increases *downward*) applies to incompressible fluids (liquids)

hydrostatic $F = \gamma h_c A$ (h_c is the vertical depth to the centroid of A)

Continuity Equation (Conservation of Mass)

$\dfrac{dM_{CV}}{dt} = (\rho Q)_{in} - (\rho Q)_{out} \qquad Q = \int_A v\, dA = VA \qquad V = \dfrac{1}{A}\int_A v\, dA$

Incompressible: $\dfrac{dS_{CV}}{dt} = Q_{in} - Q_{out}$ Steady-state: $(\rho VA)_{in} = (\rho VA)_{out}$

Momentum Equation

$\sum \vec{F}_{on\,CV} = \rho Q_{out}\vec{V}_{out} - \rho Q_{in}\vec{V}_{in}$ (used for calculating forces exerted by water)

Similitude

Reynolds # = $\left(\dfrac{\rho V L}{\mu}\right)$ = inertial F/viscous F (pipe flow, friction, boundary layer flow)

Froude # = $\left(\dfrac{V}{\sqrt{g L}}\right)$ = inertial F/gravity F (open channel flow, gravity-driven flow)

Re < ~2000 for laminar flow **Re** > ~4000 for turbulent flow

Fr < 1 for subcritical flow **Fr** > 1 for supercritical flow

Friction and Drag Equations

$$F_{Df} = C_f \, \rho \left(\frac{U^2}{2} \right) A_{shear}$$

flow parallel to a solid boundary

$$F_D = C_D \, \rho \left(\frac{U^2}{2} \right) A_{proj}$$

flow perpendicular to a 3D object

Both C_f and C_D are f(**Re**) and object shape

For *laminar* flow around a sphere we also have “Stokes’ Law”:

$$F_D = 3\pi\mu V d \qquad V_s = \frac{d^2}{18\mu}\left(\gamma_{sph} - \gamma_{fl}\right)$$

Work-Energy Equation

$$\frac{p_1}{\gamma} + z_1 + \alpha_1 \frac{V_1^{\,2}}{2g} + h_p - h_T = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^{\,2}}{2g} + \sum h_L$$

- steady flow along a fluid streamline from (1) to (2)
- head losses (h_L) include friction, and local losses at fittings, bends, valves, etc
- pumps and turbines add and extract energy, respectively
- for turbulent flow, use α = 1.0

Associated equations:

Darcy-Weisbach: $h_{Lf} = f \dfrac{L}{d} \dfrac{V^2}{2g}$ ($f = f(k_s/d, Re)$: use Moody Diagram)

Local losses: $h_{L\,loc} = K \dfrac{V^2}{2g}$ (K = empirical coeff)

Power: *power* = $\gamma Q h$ where h is any head (pump head, turbine head, head loss, etc.)

Open Channel Flow

Manning’s Equation: $Q = \dfrac{k}{n} R_h^{\,2/3} S^{1/2} A_{xs}$ (applies to uniform flow conditions)

k = 1.49 for English units, 1 for SI units

Hydraulic Radius: $R_h = \dfrac{A_{xs}}{P_w}$ where P_w is the wetted perimeter