

Coefficient Matrix for a Pinned End Condition

CE 201 Spring 2007

The boundary conditions at a pinned end are:

Deflection (y_i) = 0 - so we don't need an unknown for the pinned node

$$\text{Moment} = \left(\frac{d^2 y}{dx^2} \right)_i = \frac{y_{i-1} - 2y_i + y_{i+1}}{\Delta x^2} = 0$$

From this equation, since $y_i = 0$, we can deduce that $y_{i-1} = -y_{i+1}$

Writing the finite difference equations:

Node 1 (next to pinned end):

$$y_{\text{imaginary}} - 4(0) + 6y_1 - 4y_2 + y_3 = \frac{q_1 (\Delta x)^4}{E_1 I_1}$$

$$(-y_1) - 4(0) + 6y_1 - 4y_2 + y_3 = \frac{q_1 (\Delta x)^4}{E_1 I_1}$$

$$5y_1 - 4y_2 + y_3 = \frac{q_1 (\Delta x)^4}{E_1 I_1}$$

$$\text{Next node (2):} \quad (0) - 4y_1 + 6y_2 - 4y_3 + y_4 = \frac{q_2 (\Delta x)^4}{E_2 I_2}$$

$$\text{Next node (3):} \quad y_1 - 4y_2 + 6y_3 - 4y_4 + y_5 = \frac{q_3 (\Delta x)^4}{E_3 I_3}$$

Resulting coefficient matrix for pinned end:

	y_1	y_2	y_3	y_4	y_5	y_6
Node Next to Pinned End (1)	5	-4	1	0	0	0
Next Node (2)	-4	6	-4	1	0	0
Next Node (3)	1	-4	6	-4	1	0
Next Node (4)	0	1	-4	6	-4	1
Next Node (5)	0	0	1	-4	6	-4